

Discovery Learning with the Help of the GeoGebra Dynamic Geometry Software

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Abstract. This article gives points of view of discovery learning, the role of discovery learning and the types of the one in mathematics. Teacher will give questions so that students themselves form knowledge what teacher wants to teach through discovery acts. With the help of the GeoGebra dynamic geometry software, the process of teaching and learning becomes more effectively. Because the GeoGebra software is dynamic, it helps discovery learning a lot. This software can allow teacher and students to find out solutions of geometric problems. It also can create a lot of new problems. Especially, the GeoGebra software is free. That's reason why teacher and students should use the one. In addition, students will develop technology skills and get mathematical knowledge easily.

Keywords: Discovery learning; the GeoGebra software; problem-based learning; technology skills.

1. Introduction

In the 21st century, the creation is one of crucial factors of the process of teaching and learning. In order to create in the process of teaching and learning, teacher usually uses a familiar method being discovery learning. This method proves the activity of students. Nowadays, with the help of technology, the methods of teaching and learning are changing. With the help of technology, students have more good and effective leaning methods than traditional ones.

There are authors to research and apply technology to teaching and learning in Vietnam. For example, T.H. Trinh, he applies the Carbi software to aid

mathematical teaching and learning (Trinh, 2006). We have applied e-learning to aid mathematical teaching and learning. This article will refer to an aspect of active learning method being discovery learning with the help of the GeoGebra software.

2. Discovery learning

J. Bruner supposes that the process of learning is a subjective process (Bruner, 1961). Learners form new ideas or new concepts based on knowledge through this process themselves. Learners select and convert information, form hypotheses and give their decisions based on the structure of the process of awareness. He asserts that when learners begin to go to school, learners should know basic structures of knowledge than know data, information which are boring and these data and information force learners to memorize a lot. Students should be motivated and be discovered information freely.

By Bruner, discovery learning occurs when individuals have to use the process of thought to find out the meaningfulness of something themselves (Bruner, 1961). He gives four reasons for using discovery learning as follow: (i) To make an impulse of thought, (ii) to develop inner motivation than outer motivation, (iii) to learn the way of discovery and (iv) to develop thought (Bruner, 1961).

For the first reason: Bruner suppose that, an individual himself only can learn and develop his mind by using this method (Bruner, 1961).

For the second reason: He believes that, if teacher is successful with this method, learners feel satisfied with what they did. Students get the stimulations of intellectualness satisfactorily. Inner award is inner motivation. Teacher usually acts on outside through compliments, outer award. If teacher wants learners to find motivation or real amenities in learning, he must build methods or systems, that make learners themselves satisfactorily, than outer motivation. Internal force has the crucial role of the successfulness or unsuccessfulness of an individual learning.

For the third reason: He emphasizes that, the only way, which learners learn discovery techniques, is what they must have opportunities to discover. Learners will learn the organization and performing of their researches gradually through the discovery.

For the fourth reason: He suppose that, one of the best results of the discovery method is to aid learners' mind better, learners maintain their mind enduringly. If learners think about something which they thought and compare with given information then what learners thought is still in their mind although they learned long ago while the concepts, which learners are provided, disappeared. The documents, which are used for analyzing and going to the result, are still "fresh" in learners' mind, moreover, these documents make learners remember their forgotten concepts.

By G. Petty, there are two approaches in teaching which are : *Teaching by explaining and teaching by asking questions* (Petty, 2000).

In *teaching by explaining*, students are learned new knowledge. Students must use and memorize these new knowledge. In *teaching by asking questions*, teacher gives questions or gives exercises so that students themselves find out new knowledge (Although, there are still special guides and preparations). This new knowledge is corrected and stated by teacher. Discovery with guides is an example of this approach. Discovery learning is only used if learners can follow the new lesson from the knowledge and experiences themselves.

The author notes that, to find information in books is not a discovery learning and he emphasizes: Discovery acts are usually progressed by groups. They require learners to have high thought techniques in order to find out new things. Because learners themselves find problems, the learning has a high quality. Each of members of this group can provide “a piece of puzzle” of his understanding on the researching topic.

G. Petty also refers to the strength of the discovery method (if we use rightly) being (Petty, 2000):

It is active and it motivates students to join in the process of learning. It is high encouraged and funny. The questions make students increase their understanding and amenities for the subject. Students themselves must “find out about”, i.e, students themselves catch the problem which they are learning. As a result, learners will understand the problem, the relation between it and the pre-lesson and learners remember the lesson for a long time. This method makes students have good thought: evaluation, judgment thought, problem solving, analysis, synthesis, etc. Conversely, teacher-centered methods usually direct students to skills which rarely think such as listening-lecturing and trying to understand this lecture. By this method, students are encouraged to consider that learning is their works than experts’ works doing for them. This method allows students to learn funnily, to find out the answers for themselves, and a controversial thing is the method to develop inner motivation than outer motivation.

However, the author gives some restricts of the discovery method : slow velocity and we can not apply this method to topics such as simple topics based on real facts, or topics that students do not require to discover, etc. (Le, 2007).

3. The role of discovery learning

J. Bruner gives the advantages of using discovery learning: it motivates the innovation of thought, develops inner motivation than outer act, learners can learn discovery way and develop memories themselves (Bruner, 1961).

The process of discovery requires students to evaluate, consider, analyze, synthesize, and by Bruner, an individual only learns and develops his mind if he uses it (Bruner, 1961). On the other hand, when he achieves some results in the process of learning, the learner feels satisfied what he did and he will have a desire to tend to difficult works, this is inner motivation.

G. Polya, who is a famous mathematician and educator, supposes that, *The major task of teaching mathematics at school is to teach students thinking* (Polya, 1979). By him, in order to get the best effective learning, students themselves need to *discover in moderation* with a large of number of their documents. By Lichtenbe (The German physician): “what he force to discover himself leaves in his unconscious a small path that he can use it if he needs” (Polya, 1979).

D. Ausubel considers “discovery learning is an excellent one to create the way of the synthetic learning” (Petty, 2000). In addition, educators suppose that, discovery learning shows the strong-points as follow:

+ *It is student-centered method.*

One of principles of learning shows that, if the more learners participate in the process of learning a lot, the more they learn a lot. When teacher thinks about learning concept, they usually suppose that learners are only people getting knowledge than people processing information. This is a restricted view on learners. Actually, learning must include all of aspects in order to make learners have all abilities.

For example, in discovery situations, learners do not only comprehend concepts, rules but also learn the way of the build of their directions, responsibilities and communication in the social. In teacher-centered method, a lot of the above abilities are not for students. If we find out a guide that make students have all of the above abilities, the learning environment is not a total of teacher-centered method.

+ *It is a method to make each of learners developing his aware competence*

All of us have aware competence ourselves. If these competences] are good, we feel safe on psychology (confidence). We easily get new knowledge, readily get chances, find out about and accept everything even the unsuccessfulness, we become more creatively, have a good thought competence and become the men who work effectively. A part of task becoming effective men is to build aware abilities of ourselves. We only do this thing by attending discovery acts and finding out about these acts. Because of attending these acts, we have a deep view on our competence. Discovery method makes learners have a lot of big opportunities attending learning acts, from that point, learners have a deep view on competence, build aware competence themselves.

4. The way of becoming a man-owned

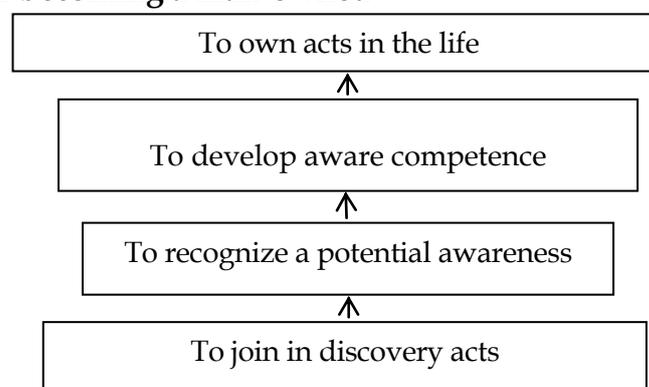


Figure 1. The way of becoming a man-owned

+ *The required level is increasing*

One other aspect of self of a man is his required level. For example, will a child know his competence of how he completes a work? Many children accept a low required level being what teacher usually listen either "I can not learn mathematics" or "I have never been good at science subjects", etc. However, if children join in discovery acts, they will learn the way of independent thought. In other words, by having successful experiences of competence themselves, they will see that "they themselves can solve this problem without any help from teacher, parents or anyone". This thing follows "I can do everything". From that point, the required levels of their competence are themselves improved.

+ *The method develops talents*

The talent on learning concerns with one of talents of everybody. The more we are free, the more we have opportunities to develop that talent. For example, when children work together to find out about something, it means that they join in the process of talent development each other such as : to make a plan, to organize, to communicate, to think creatively, to make the competence of learning.

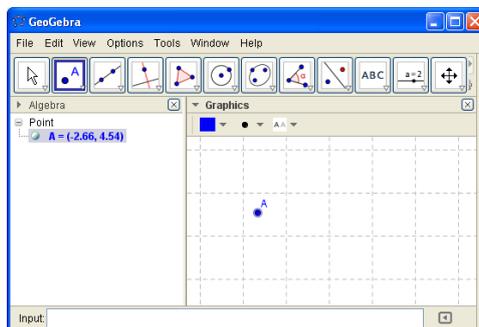
+ *The method allows learners to get and update information*

Teacher is usually in a hurry of his teaching while learners need time to think and use their mind to reason and to find out about concepts, rules deeply. If information becomes a part of learners' mind with a particular meaning, learners need time to do that. Jean Piaget asserts that : There is not a precise learning's concept unless learners have time to reason about achieved information and through the process of that thought, learners acquire and update what learners met in particular situations.

5. Some tools in the GeoGebra software

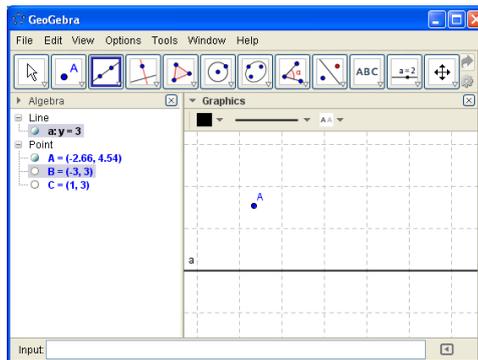
5_1. Point

Use the tool **Point** to draw a point A .



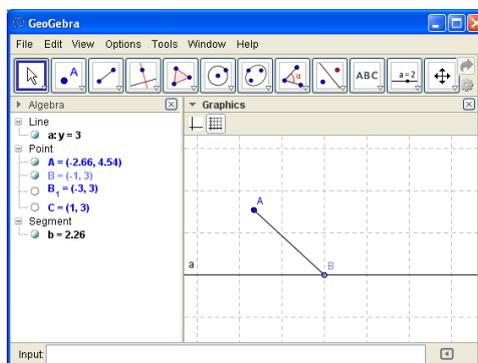
5_2. Line

Use the tool **Line** to draw a line a .



5_3. Segment

Use the tool **Segment** to draw a segment AB .



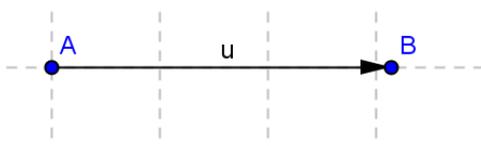
5_4. Ray

Use the tool **Ray** to draw a ray from A to B .



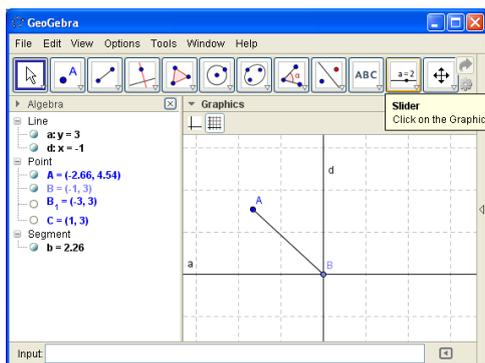
5_5. Vector

Use the tool **Vector** to draw a vector from A to B .



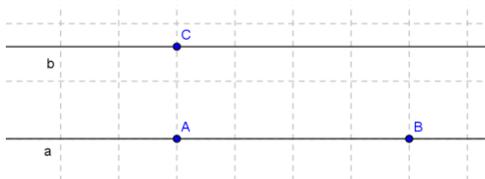
5_6. Perpendicular Line

Use the tool **Perpendicular Line** to draw a perpendicular line d to a .



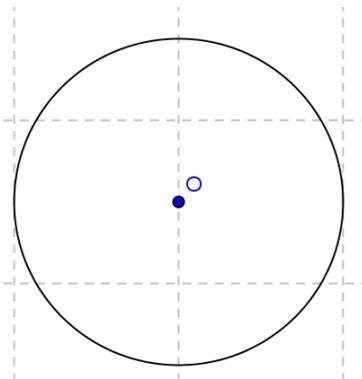
5_7. Parallel Line

Given a line AB . Draw a line b passing through a given point C , which is parallel to AB : **Parallel Line** $\rightarrow C \rightarrow AB$.



5_8. Circle with Center Through Point

Use the tool **Circle with Center Through Point** to draw a circle with center O and radius r .

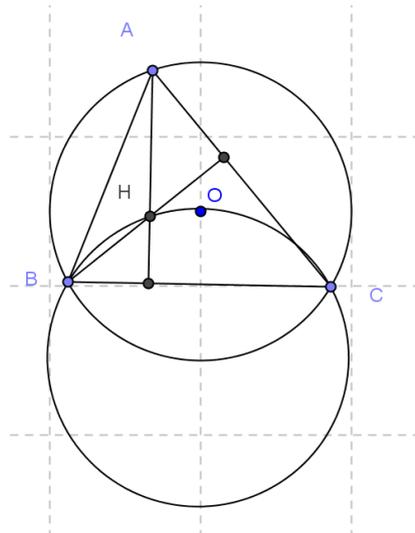


5_9. Locus

Use the tool **Locus** to find the locus of a movable object.

For example, let (O) be a circumcircle of a given triangle ABC . Suppose B, C are fixed points and A is movable point on this circle. Let H be the orthocenter of the triangle. Find the locus of the point H .

We find the locus of the point H as follow:



- +) Construct the circle (O) .
- +) Construct three points A, B, C .
- +) Construct the triangle ABC and its orthocenter H .
- +) Locus $\rightarrow H \rightarrow A$, we get the locus of the point H being a circle as the above figure.

6. Some main characteristics of the GeoGebra software

- +) The GeoGebra software helps the users to find the locus of an object. (See 5_9).
- +) It helps the users to predict the results of some extremum geometric problems.
- +) It is very effective in solving, teaching and learning the analytic geometry.
- +) Since the GeoGebra is a dynamic software, it is very useful in verifying geometric problems.
- +) It helps teacher a lot in teaching definition, theorem, solving problems.

7. Forms (levels) of discovery learning

Depending on whether teacher intervenes the process of discovery of students or not, we divide discovery acts into two types: Guided discovery and free discovery.

In guided discovery, teacher gives problem, provides context, necessary tools and students have opportunities to discover, solve problem. Teacher here plays a role as an encouraging, assistant man to ensure that students do not get troubles or do not perform their surveys, experiments. However, teacher's help

should be formed in questions to help students to think about the process of discovery but is not showing student what they need to do.

Guided discovery has two levels : total guide or partial guide.

After students join in discovery acts with guides, they are ready for free discovery acts. In these discovery acts, students themselves must define what they want to research, must select their way, solution and must research until they find the result.

Table 1. The levels of problem-based learning and discovery learning

Level	Teacher's acts	Students' acts	Problem-based learning	Discovery learning
1	To give acts so that students do these acts	To do according to teacher's instructions in order to achieve goals	To give a lecture in order to find out and solve the problem	To discover with total guide
2	To give a problem, not to give its solution	To find out the solution themselves	To make oral questions in order to find out and solve the problem	To discover with partial guide
3	To choose the start situation or accept students' choices	To define problem in the situation, to find out the solution themselves	To perform a self-research on the problem	To discover freely

(Le, 2007).

8. Example

A river has straight parallel sides and villages A and B lie on opposite sides of the river (see Fig 2). Where should we build a bridge MN in order to minimize the travelling distance between A to M and B to N (a bridge, of course, must be perpendicular to the sides of the rivers) ?

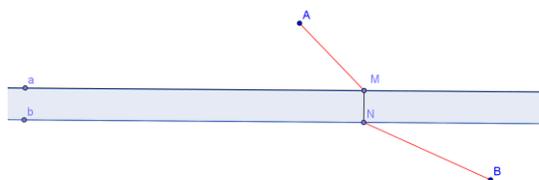


Figure 2. A river having straight parallel sides

Teacher gives discovery acts being guided questions for students with the help of the GeoGebra software.

Teacher uses the GeoGebra software in order to help students to find out the solution. Teacher constructs and finds out the position of the bridge MN such that $AM + BN$ is minimal on the GeoGebra software as follow:

Step 1. Construction

- . Draw a ray Dx' .
- . Take a point F on the ray Dx' .
- . Draw two parallel lines a and b such that the distance of a and b is equal to DF .
- . Draw movable point M on a .
- . Draw a straight line passing through point M and perpendicular to the straight line a , cut the straight line b at a point N .
- . Draw the Cartesian coordinate system Oxy such that MN is perpendicular to Ox .
- . Take point T on Oy such that $OT = AM + NB$.
- . Draw a straight line passing through point T and perpendicular to Oy , cut the straight line passing through point M and perpendicular to Ox at a point E .
- . Hide unnecessary objects.

Step 2. Making trace

- . Making trace to point E , moving point M , we obtain the trace of point E .

Moving point M to the position such that the ordinate of point E is minimal, we determine positions M, N that need to be found (see Fig 3).

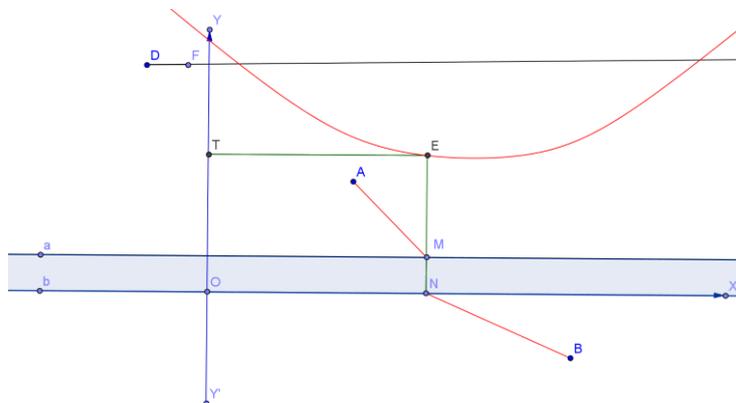


Figure 3. Finding out the position of the bridge MN on GeoGebra

Teacher : The directions of d are the lines being parallel or coincident with d . Hence, which directions of line are invariant?

Students : Directions of line MN are invariant (because MN is always perpendicular to river shores a and b).

Teacher : Because directions of line MN are invariant, we will use a translation of directions MN . Because MN is constant, vector \overline{MN} is constant. Using the translation $T_{\overline{MN}}$, we have $A \mapsto A'$. Then, what is the image of point M ? Which segment is equal to AM ?

Students : $M \mapsto N$ and $AM = A'N$.

Teacher : Hence $AM + BN = A'N + BN \geq A'B$. The equality occurs when and only when N is the point of intersection of $A'B$ and b (by triangle inequality). Do you draw points M, N ?

Students : We have the constructions of points M, N as follow :

Using the translation $T_{\overline{CD}}$ (\overline{CD} is a vector which its directions are perpendicular to river shores, the length of CD is equal to the width of the river, $C \in a, D \in b$), we have $A \mapsto A'$. $A'N$ meets b at N . Using the translation $T_{\overline{DC}}$, we have $N \mapsto M$. Then MN is the bridge that needs to be constructed (see Fig 4).

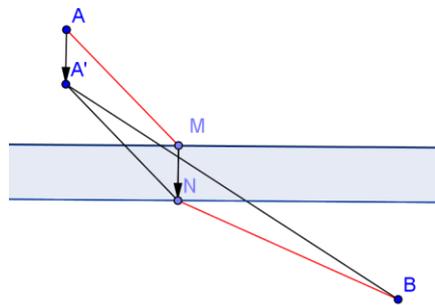


Figure 4. The drawing of points M and N

Teacher gives remarks : The result, which we find out on the GeoGebra software, is absolutely suitable with the mathematical result.

Teacher allows students themselves to discover knowledge on the GeoGebra software by moving, drawing additional lines. These acts are controlled by teacher.

9. Results and Discussion

We delivered survey forms to 37 teachers of upper secondary schools in Ho Chi Minh city, Viet Nam in order to check the suitability of discovery learning with the help of GeoGebra :

**Table 2. The ideas of teachers on discovery learning
with the help of the GeoGebra software**

Survey contents	A total of teachers	Ideas of teachers	
		Agree	Disagree
To help teacher to teach mathematics	37	34	03
To help students themselves to learn mathematics	37	34	03
To improve the interaction between teacher and students	37	35	02
Teacher interests in technology	37	23	14

Through the surveys of the ideas of teachers shown at the table 2, we first assert that discovery learning with the help of GeoGebra is suitable with the self-learning of students and it improves the effectiveness of teaching and learning basically.

We also delivered survey forms to 282 students of upper secondary schools in Ho Chi Minh city, Viet Nam on the exploitation and uses of the GeoGebra software for the help of discovery learning. We obtain good results as follow :

**Table 3. The ideas of students on discovery learning
with the help of the GeoGebra software**

Survey content	A total of students	Ideas of students		
		Frequently	Sometimes	Never
To perform a self-research on the GeoGebra software	282	199	61	22
To use GeoGebra in self-learning according to teacher's requests	282	243	29	10
To use GeoGebra to do exercises	282	220	43	19

To use GeoGebra in self-check of students' knowledge	282	223	44	15
To use GeoGebra familiarly	282	237	24	21

Table 3 shows that most students joined in the learning according to discovery learning with the help of the GeoGebra usually interest and exploit the features of this software in the process of their learning in order to improve their understanding of lesson and exercises. The figures shown in the table 3 showed that the rate of the self-learning of students with the help of the GeoGebra software is high. We can assert that discovery learning with the help of GeoGebra acts on students well. This thing proves that this method makes students pleasant and interested in learning mathematics. This method is more effectively than traditional learning methods.

Table 3 also shows that the GeoGebra software helps teacher to teach mathematics. Teacher can use this one for finding out the results of problems, for verifying problems. In addition, it also gives teacher to create new problems that their results are known immediately. This software is effective in teaching geometric concepts, theorems and definitions. The processing of teaching and learning is improved. The interaction between teacher and students becomes well. Finally, the level of informatics of teacher improves gradually. Teacher likes to find about the applying informatics in mathematics.

We and students discuss on the usefulness of the GeoGebra software. These students show themselves to like this one. With multiple tools and the accurate numbers of the software being very high, the students suppose that they can solve problems with the help of the software easily. Since the help of the one, they even do not need the help from teacher. When they use this software, they become more creatively. They can create new problems and prove these problems.

10. Conclusion

After students learned according to discovery learning, we realize that: Students write well, briefly. They also express mathematical languages coherently and fluently. This thing proves that the competence of the self-learning of students develops clearly. Students are interested in mathematical learning. They develop mathematical thought specially to develop the creative thought. Students also improve technology skills clearly and like to research the knowledge of the GeoGebra software. The effectiveness of the new method is better than traditional methods

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